M307 Intro to Abstract Mathematics – Spring 2021

Instructor information:

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Office hours: Monday 11-12 (by Zoom); Tuesday 1-2 (by Zoom); Wednesday 10-11 (in person); Friday 2-3 (in person); or by appointment.

Our class will meet over Zoom (Meeting ID: 991 0861 6990, Password: Abstract) at 12 noon, MWF. We'll use this same Meeting ID and Password for the Zoom office hours.

Course description: At its core, this is a course in writing and creative problem solving. Our goal is to develop our skills to write and communicate effective proofs of mathematical facts. You will build on your skills for reading and writing mathematics and work to familiarize yourself with the abstract reasoning necessary in upper division mathematics courses.

The skills you will develop in this course are highly valued in the world outside math. No matter the profession you ultimately choose, one of the most important ways you will be distinguished from your peers will be your ability to communicate effectively and to think clearly, critically, and abstractly.

We will learn according to an educational philosophy called 'inquiry-based learning' (IBL) in order to encourage you to take a very active personal role in your learning. The IBL philosophy suggests that students should be responsible, as much as possible, for guiding the acquisition of knowledge and validating the ideas presented.

Learning Outcomes:
Upon completion of this course, students will be able to:

1. Recognize correct and incorrect mathematical reasoning.
2. Demonstrate an understanding of basic mathematical logic.
3. Use counter examples in rigorous arguments.
4. Write clear and concise proofs using the following methods: direct proof, contrapositive proof, proof by contradiction and proof by mathematical induction.
5. Write clear and concise proofs using basic set theory.
6. Write clear and concise proofs using precise definitions for functions and relations.
7. Write clear and concise proofs using injections, surjections, and bijections to compare cardinalities of sets.

Textbook: We will not use a formal text in this class. Instead we will follow a list of problems and tasks adapted for IBL and based on a set of notes written for this purpose by Prof. Dana Ernst at NAU. You will be expected to read and digest the material from these notes. You should be looking for clarification whenever necessary by asking questions.

Read your math slowly and with a piece of paper and a pen/pencil nearby. Use this paper, not only to take notes, but also to figure stuff out, look at examples, and ask your own questions.

Course Structure:

Daily homework

In each class period, you will be assigned between three and eight tasks which need to be completed before the next class meeting. You should work in pairs to complete these tasks; I will use a random number generator to assign the pairs, and the pairings will change weekly. Your pair's solutions to the daily homework should be carefully, clearly, and cleanly written in order to receive credit. You should work in drafts, and hand in your final draft either on Moodle, or by emailing a photo/PDF of the homework to me before class starts. (One email per pair is sufficient, but make sure either the email or the homework has both names on it.)

The daily homework in this class will usually not be a simple or straightforward application of concepts that have previously been explained to you. Rather, you will have to think hard and creatively to see how the tools you've developed can be used to solve the homework problems. Often, you will learn new concepts by doing the homework. Once you have worked hard on the material before class (by reading the notes and working on the homework), we will spend class time clarifying everyone's understanding through your presentations.

Presentations

We will spend most of each class period having you, the students, present your solutions to the Daily Homework to the rest of the class, and polishing/revising those solutions until everyone in the class is convinced that the solution is watertight. I will select presenters based on (computer) random choices.

I expect that each day you will come to class feeling at least 90% confident about your solutions to that day's Daily Homework problems. The nature of this class means that you will not always be right, though! That's OK. Making mistakes (and learning how to fix them) is a crucial part of learning. Correcting a mistake in class, where everyone can learn from it, is also much more efficient than correcting it on each person's homework.

With that in mind, we will have a class document of *Proof-Writing Dos and Don'ts*. We'll start building it on the first day of class and update it throughout the semester. I expect your presentations to avoid the flaws indicated in the current version of *Proof-Writing Dos and Don'ts*. But if you make a new mistake, it won't affect your grade.

Each presentation will be graded out of 4 points, according to the rubric below.

> **4:** Presentation is polished, clear, and correct according to the current version of *Proof-Writing Dos and Don'ts*.
>
> **3:** Presenter makes a few mistakes, or has misunderstood part of the problem, but the class' input enables them to quickly bring the solution up to Level 4 quality.
>
> **2:** While there's some good stuff here, there are too many mistakes and misunderstandings for us to reasonably correct in class.

1: Minimal progress.

0: Chosen presenter is absent/No preparation is evident.

To make sure we're using class time effectively, if your presentation is a Level 2 or Level 1, we will not work on polishing it as a class. Instead, I will point out the flaws and then ask if anyone has a different approach to the presentation that they want to present. Assuming the latter is a Level 3 or 4 presentation, after polishing, we'll compare it to the original one and try to identify takeaways for the *Proof-Writing Dos and Don'ts.*

Class Participation

The main goals of this class are that each of you will learn to distinguish between correct and incorrect mathematical reasoning, and to produce quality mathematical writing. Consequently, after each presentation, we will discuss the presented solution as a class. The focus of this discussion will be to (a) perfect the given presentation; and (b) enhance everyone's understanding of the underlying mathematical reasoning.

I expect everyone to participate actively in the class discussion following each presentation. In addition to comments like "I'm not sure if your argument in Step 2 is precise enough, because XYZ" or "Shouldn't the symbol n mean the same thing throughout the proof?" please also ask questions like "How did you come up with that equation?" or "Did you think about X approach?" or "How do we know Y?" If your solution is different from the one presented, they might both be correct! Let's try to figure that out.

I also expect you to participate respectfully in the class discussion. More on that later.

Each week, you can earn up to 1 point in the "Class Participation" category. If you contribute to the discussion each day, you'll earn the point; if you speak up once in the week, you'll earn 0.5 points. (You won't earn any points for a comment that boils down to "I agree with what's already been said.")

So that I can ensure everyone has a chance to contribute to the discussion, I'll ask you to use the "Raise Hand" feature in Zoom and wait to be called on.

Weekly homework

Choose 2 problems from the previous week's Daily Homework to write up beautifully. Use the LaTex template posted on Moodle, and submit the final PDF via the assignment link in Moodle. Each problem will be graded on the following scale:

1. Correct and well-written. 4 points.
2. Good work but some mathematical or writing errors that need addressing. 3 points.
3. Some good intuition, but there is at least one serious flaw. 2 points.
4. I don't understand this, but I see that you did work on it. 1 point.
5. No work is evident. 0 points.

Exams

We will have two midterm exams and a cumulative final exam. Each midterm with be worth 10% of your final grade and the final exam will be worth 15%. All exams in this class will be oral exams. A week before the exam date, I will give you a list of approximately 6 problems that could appear on your exam. You must work independently to prepare the solutions to the exam problems. On exam day, you will schedule a 20-minute meeting with me in which I will randomly choose one of the exam problems, for you to present to me, without hints and without notes.

Midterms are tentatively scheduled for Thursday February 18 and Thursday March 25. Our final exam date is Thursday April 29.

Grade breakdown

6. Homework (daily and written): 30%
7. Class participation: 5%
8. Presentations: 30%
9. Midterm exams: 20% total
10. Final exam: 15%

Rules:

- Participation is a critical part of this course and your attendance is required.
- On the daily homework, intra-class collaboration is encouraged. Indeed, you will do the Daily Homework in pairs. However, everyone will write their own weekly Written Homework.
- The homework you turn in must be in your own words and reflect your own understanding. The goal of the homework assignments is (1) for you to strengthen the "mental muscles" needed in upper division math courses; and (2) for me to see and correct any mistakes you're making before they become an unbreakable habit (and before the exam). The more you rely on others to help you with the homework, the less effective it is at achieving these goals.
- Late homework will generally not be accepted. Homework turned in after class is considered late. However, to give everyone a buffer for busy weeks, everyone starts the semester with 20 points extra in the "Homework" category. (Most of the daily homework assignments will be worth 3-5 points -- one point per problem, graded on completion -- and the weekly written homework assignments will generally be worth 8 points -- 4 points per problem.)
- When working on the homework and exams for this class, **No outside resources are allowed.** This includes the internet, other texts or notes, and people who are not taking M 307 this semester.

Notable Challenges

- You will routinely be asked to think deeply about new ideas.

- You will constantly be asked to work on problems which you have not been shown how to solve.
- You will be challenged directly about the details in your reasoning.
- You will be criticized in your writing.
- You may experience frustration and failure before success. Becoming comfortable with this process is one of the most empowering skills you can develop to become an independent lifelong learner.
- Solving new problems and mastering new concepts is both difficult and time consuming. It may take time, experimentation, and deep thinking before you develop a plan for how to even begin a task.

Course guidelines and policies:

Classroom and Course-related Behavior

University policy requires that all of us in the classroom treat each other with respect, and refrain from behavior that will disrupt the educational process. Please refrain from using any electronics during class that are not directly related to what we are doing.

If you would prefer to be called by a different name, or gender pronoun, than is listed on the course roster, please let me know. I'm happy to make that adjustment.

Digital Access

Digital devices (like laptops and cell phones) are becoming increasingly important to success in college. In this course, you may need digital devices to access readings, complete and submit written assignments, complete online quizzes, verify your attendance, take in-class polls, coordinate with other students regarding group projects, complete and submit group projects. I recognize that some students are unable to afford the cost of purchasing digital devices and that other students rely on older, more problem-prone devices that frequently break down or become unusable. I also recognize that those technology problems can be a significant source of stress for students. Given those challenges, please contact me if you experience a technology-related problem that interferes with your work in this course. I am more than happy to help you in accessing support.

Student Conduct Code

All students need to be familiar with the Student Conduct Code. You can find it at http://www.umt.edu/student-affairs/dean-of-students/default.php or by searching in the "A to Z Index" on the UM home page. In particular, discrimination and harassment are not tolerated at the University of Montana. If you feel that you have been subjected to discriminatory or harassing behavior, please contact the Office of Equal Opportunity and Affirmative Action at 243-5710 or http://www.umt.edu/policies/browse/personnel/discrimination-harassment-sexual-misconduct-stalking-and-retaliation for help in addressing the situation. You can also report the discrimination or harassment to me or to another faculty member you trust.

Academic Honesty

I take academic honesty very seriously and I will act on any transgressions that I notice. Misconduct is subject to an academic penalty in this course and/or a disciplinary sanction by the university. We all know that a record of academic misconduct is a very bad thing to have documented in your academic history.

Disability modifications

The University of Montana assures equal access to instruction through collaboration between students with disabilities, instructors, and Disability Services for Students. If you think you may

have a disability adversely affecting your academic performance, and you have not already registered with Disability Services, please contact Disability Services in Lommasson Center 154 or call 406.243.2243. I will work with you and Disability Services to provide an appropriate modification.