

Math 273 - Multivariable Calculus

Fall 2017

Course Instructor: Dr. Elizabeth Gillaspay

Contact Info: Office: MATH 012, e-mail: elizabeth.gillaspay@mso.umt.edu.

Lecture hours and Venue: MTWF 9:00 a.m. - 9:50 a.m., MATH 103

Final Exam: Thursday, December 14, 8:00 a.m. - 10:00 a.m., MATH 103.

Office Hours: MWF 10 - 10:30 AM; Monday, 4:30 - 5:30 PM; Thursday, 1:30 - 2:30 PM; or by appointment.

Course Prerequisites: Math 172 (Calculus II)

Textbook: Gleason, Hughes-Hallett, McCallum, et al, *Calculus: Multivariable*, 6th edition.

Learning Assistant: Ryan Smith

Contact Info: rs110587@umconnect.umt.edu

Ryan's Office hours and Venue: Tuesdays, 10:00 a.m. - 10:50 a.m., MATH 212

1 Course Contents

Welcome to Math 273, Multivariable Calculus. This course is by far my favorite of the calculus courses; in fact, it's one of my favorite courses in the math major.

In this class, you'll learn how to extend the big ideas from calculus to functions of more than one variable. To do this, you'll have to begin to think in 3 and 4 dimensions. This will be a challenge at first, but I really enjoy it and I hope you will too.

Content-wise, the goal of M 273 – the big main result that we will spend all semester building up to – is a collection of higher-dimensional analogues of the Fundamental Theorem of Calculus. Known as Green's Theorem, Stokes' Theorem, and the Divergence Theorem, these theorems explain how integration and differentiation relate for different types of functions of multiple variables.¹

I also have another goal for us this semester, which I have for every class I teach: I want you to improve your ability to communicate math, not just to enhance your mathematical knowledge. Because of this, M 273 will also include presentations of homework problems; reading the textbook; and working in groups.

1.1 Learning outcomes

Upon completion of this course, a student will be able to:

¹In fact, all three theorems, and also the Fundamental Theorem of Calculus, are a special case of an even more general theorem, known (confusingly) as Stokes' Theorem, which you will learn about if you take a course on Multivariable Analysis or Differential Geometry.

1. Compute dot and cross products, and work with equations of lines, planes, cylinders and quadric surfaces;
2. Find formulas for vector-valued functions and space curves, and compute their derivatives, arc length, curvature and motion;
3. Compute limits, partial derivatives, directional derivatives and gradient vectors of functions of several variables;
4. Find tangent planes to surfaces, and linear approximations;
5. Use the chain rule, find extreme values, and solve constrained optimization problems with Lagrange multipliers;
6. Compute double and triple integrals, including in polar, cylindrical and spherical coordinates;
7. Compute line integrals and use the Fundamental Theorem of Line Integrals;
8. Use and understand Green's Theorem, and curl and divergence of vector fields;
9. Calculate surface integrals, and use Stokes Theorem and the Divergence Theorem.

Through this course, students will also improve their ability to read and communicate mathematics, orally and in writing.

2 Course Structure

Your grade in this course will be based on:

- WeBWorK (15%)
- Discussion posts (8%)
- Written homework [completion/writing] (15%)
- Quizzams/Quests (25%)
- Homework presentations (12%)
- Final exam (25%)

For each hour of class time, you should expect to spend approximately 3 hours outside of class working on material related to Math 273.

My plan for the structure of the average class period is the following. I will spend a few minutes introducing the topic of the day, and then you will spend most of the hour doing some activities to develop your familiarity with the topic. Then, you will have some homework to do before the next class period, to check your understanding of the topic. Usually, that homework will take the form of reading the relevant section in the textbook; solving several related WeBWorK problems; and posting in the relevant Discussion forum in Moodle to tell

me what aspects of the topic you want to discuss further in class the next day. What do you still find confusing? What would you like to learn more about? What questions came up as you were reading or doing the WeBWorK? This homework (both WeBWorK and the Discussion posting) will usually be due by 7 AM the morning of the next class meeting.

You will be allowed multiple attempts on the WeBWorK problems.

Tuesdays will **not** be “average class days;” see Section 2.4 below for a description of the average Tuesday.

In addition to the daily homework of reading the textbook and WeBWork (which I anticipate will take you about 2 hours each time, for a total of 6 hours per week), you will have one written homework assignment every 2 weeks. These written homework assignments will be the basis for the Quizzams² and the homework presentations, which are described in more detail below. You should anticipate spending **8 hours** over the course of the 2 weeks to solve the problems on the written homework assignment.

Since M 273 is a 4-credit course, this leaves you with 4 hours in each 2-week block to spend on extra practice problems, studying for the Quizzams, and preparing your homework presentations.

2.1 Reading the textbook

In order to have enough time in class to ask and answer questions, to make sure that you fully understand the material, you will have to learn some of the basic ideas and definitions outside of class, via the reading assignments.

As mentioned above, I will ask you to read a section of the textbook before most class meetings. Although I will spend some time in class the previous day introducing the section, sometimes these reading assignments will be your first introduction to a concept. When this happens, it’s not a scheduling mistake on my part, but a chance for you to practice learning math from the textbook instead of from lectures.

If you don’t understand everything you read in the textbook, don’t worry! Please ask me about it, via the Discussion forums on Moodle, in class, or in office hours. Reading a math textbook is very different from reading for a history class or a political science class. It’s a skill that we will spend time developing, both by discussing (in class) strategies for reading math, and by regular practice with the reading assignments. My goal is that by the end of the semester, you’ll be comfortable learning math from the textbook.

2.2 WeBWorK (15%)

The goal of the WeBWorK homework is for you (and me) to see what **you** understand – not what your friend understands or your textbook understands, or what the Internet understands. For that reason, the Honor Code requires that you make a first attempt at the WeBWorK exercises with only the following resources:

- A pen or pencil
- A blank piece of paper
- Your brain.

²A *Quizzam* (or *Quest*) is an assessment that has the frequency of a quiz but the seriousness of an exam.

If you can't quite solve the problem with just these resources, notice where you get stuck – this means you need more practice with this type of problem. Can you articulate what, exactly, you find confusing? The more specific you can be, the better you can focus your studying.

If you get stuck on a problem, you can ask for a hint – from me, from your friends, from the textbook. However, **do not** let someone else solve the problem for you! As soon as you have read or heard enough to get a new idea (even a tiny one) for how to approach the problem, **walk away** (or close your book, or your internet browser) and try again with just your brain, a pen, and a piece of paper. Repeat this process (make an effort, look for a small hint, try again) as often as you need to.

Doing these “mental pushups” on a daily basis is necessary if you want to do well in this course. MATH 273 will require you to use your brain in new and different ways; three dimensions are much more complicated than two. To develop your ability to think in a new way, you have to work at it! Just as with weight lifting, to make your brain stronger you need to make it do as much work as possible, every day. It's OK if you can only do two or three “mental pushups” at the beginning of the semester if you make sure that you're doing as many as you can, every day, then by the end of the semester, your brain will be strong enough that the final exam will seem easy.

2.3 Discussion posts (8%)

Once you've read the textbook and done the WeBWorK assignment for a given section, I expect you to post in the relevant Discussion forum in Moodle to summarize your understanding of the material and/or tell me what aspects of the topic you want to discuss further in class the next day. What do you still find confusing? What would you like to learn more about? What questions came up as you were reading or doing the homework?

By summarizing for me your understanding of the material, or telling me the aspects of the reading assignment that you found most confusing, these Discussion postings will help me to structure class time so that I can correct any misconceptions, clarify confusing points, and skip over concepts that everyone understands.

The discussion posts are a means for you to synthesize what you've learned, connect the material to previous sections, and let me know what you still need some help with. Your comments in the Discussion forums will be graded on a scale of 0 to 2, based on how well you convince me that you have read the section and thought about it, and I will drop your lowest 5 scores.

For example:

- You will receive a 0 if you don't post any comment.
- A comment like “Everything makes sense, I have no questions” will earn a score of 0.
(Even if everything makes sense, I'm sure you can find something more specific to say! How does this section connect with other sections? What did you think was interesting? Was there any part of the section that didn't make sense at first? What was it that eventually made that part make sense?)
- If your comment asks a question that indicates to me that you only skimmed the textbook, you will earn a score of 1.

For example: “Why are we focusing only on cross-sections where the x value is held constant?”

(In the definition of a cross-section on page 676, cross-sections with y fixed are also discussed, although in less detail.)

- A comment such as “Will we be studying functions of more than two variables in this class? How might one draw four-dimensional space?” will earn a score of 2.

These questions demonstrate that the author read the section and thought about what s/he read.

2.4 Tuesday worksheets

On Tuesdays (starting September 12) we will spend class time, in groups, working through an activity that will either solidify your understanding of concepts we’ve recently introduced, or prepare you for the next section. Ryan Smith, our Learning Assistant for Math 273, will attend class on Tuesdays to help answer questions that come up as you’re working on these activities. Ryan will also have office hours in the Student Lounge (Math 212) on Tuesdays from 10 to 11 AM.

These worksheets will be an integral part of the course! Through them, you will learn material that will not necessarily be covered in lecture or by reading the textbook, and problems from the worksheets may show up on the Quizzams or the final.

2.5 Written homework, presentations, Quizzams

In addition to the daily homework described above, we will have **biweekly written homework assignments**. The Quizzams and homework presentations will also be based directly on these homework assignments, so the written homework assignments will be the basis for **55%** of your grade in this course.

Both presentations of homework problems, and group assignments, will be an integral part of this class. When you solve a problem in a group (of, say, 3 people), you usually end up understanding it from 3 different perspectives, so you have a much more thorough comprehension of the material. Explaining something to another person is also an excellent way to check that you really understand the material. Plus, the communication and teamwork skills that you develop via these activities will be useful in any career.

Between the first and second day of class, you will be divided into groups of 3 or 4. You will work with this group on the homework and worksheets for the first six weeks of the semester; we will rotate the groups on October 13.

Every second Friday (starting September 1) you will receive a homework assignment, consisting of problems that are more complicated or longer than the WeBWorK exercises. Although I encourage you to work with your group on solving the homework problems, **each person should turn in one set of solutions** to this homework two weeks later, at the beginning of class on Friday, and it will be graded on completion and on writing. Still, you will need to make sure you can solve all the problems on each homework assignment! Otherwise you will be inadequately prepared for the homework presentations and the Quizzams.

Therefore, I expect you to treat the homework assignments seriously. In particular, please:

- Start early!

For one thing, you will be expected to present one or two problems from the homework in class on the Wednesday before it's due (see the section on "Homework presentations" below). For another thing, it can be tricky to schedule time with all of your group members to work on the homework. The earlier you start, the easier it is to reschedule a meeting if necessary (though I recommend **scheduling regular group meeting times**, twice per week, to work on the homework).

I will not be available to answer questions about the homework on Friday morning at 1 AM. I will be available to answer questions during office hours earlier in the week. I recommend trying to **finish the homework by Thursday afternoon** so that you have enough time to ask me any last-minute questions, and write up the solutions to turn in. Remember that the written homework should take you **8 hours** to complete; please budget your time accordingly. I recommend trying to complete the relevant problems on the written homework within 2 to 3 days of when we introduce that section in class.

- Write your solutions in complete sentences.

I will give you some examples of what this means in a mathematical context; your textbook is also a good model for writing mathematical solutions in complete sentences.

On the first homework assignment, I will mark places where your writing is unclear or not in complete sentences, but I will not take points off for this until the second homework assignment.

- Do not start writing up the version of the homework you plan to hand in until you completely understand the answer!

As with any writing assignment, you should **write at least one rough draft** before writing the final version. In this case, the rough draft might be written on scratch paper, or on the board, during your discussions with your group-mates.

Often, the best way to explain the solution is not the same as the thought process you followed in order to discover the solution! After you have found the solution, pause and take a step back. What are the really important steps to the solution? What's the most logical order in which to do them? Write yourself an **outline** indicating this, and save it with your notes on that problem. Use this outline when you're writing up the solution.

- Work together with your group, and/or with other groups, to figure out the answers to the homework problems, but write up the solutions for turning in individually.

When writing up the homework solutions, please use the same procedure as for the WeB-WorK problems: Start with just a pen/pencil, a piece of paper, and your brain. If you discover that you can't remember how the solution went, get a **small** hint from your notes, a classmate, the textbook, a tutor, etc. Once you see the next step in the solution, close the textbook/put away your notes/etc and keep writing. In other words, **write up the homework solutions in exam conditions** or as close as is reasonable.

If you do this, then you're doing more of the "mental pushups" that will improve your understanding of Multivariable Calculus and help you to perform well on the Quizzams

and the final exam. Copying down solutions that you don't understand will not prepare you for the Quizzams or the final!

- Solve each problem to the best of your ability.

The goal of the homework is to let you practice the techniques we learn in class, and to help you – and me – identify any struggles and misconceptions that you have with the course material, so that we can address them. Turning in a “solution” that you know is garbage is thus a waste of your time and mine.

You will ALWAYS lose points on the homework by turning in nonsense. However, if you write down a partial solution that indicates where and why you got stuck, you will get full credit. (You will lose some points on the Quizzams and the final for incomplete solutions, but there as well, you will lose many more points if you write nonsense.)

Failure to follow these guidelines may result in a loss of points on the homework assignment. It will definitely mean that you are insufficiently prepared for homework presentations and Quizzams.

Grading for “Written homework [completion/writing] (15%)”:

Except for the first homework assignment, each of the six homework assignments will be graded out of 5 points; 3 points for completion and 2 points for quality of writing. You will lose points if your homework is not written in complete sentences, includes nonsensical or “garbage” answers, or otherwise fails to meet the guidelines above.

The first homework assignment will be graded out of 3 points, only on completion. I will mark places where your writing needs to improve, but I won't grade for quality of writing on the first homework assignment.

Thus, there will be 28 possible points in the “Written homework [completion/writing]” category. In addition to the six biweekly homework assignments, there will also be a final review homework assignment (not tied to a quiz) which will contribute at least 3 points to this category. Thus, in effect, you can lose 3 “completion” points on the homework assignments without any negative impact on your grade.

However, for each additional 2 points you miss on the written homework assignments, **your final grade will drop by 1%**, in addition to the impact this lack of preparation will have on your Quizzam scores. I repeat: please take the written homework seriously.

2.5.1 Quizzams/Quests (25%)

Every other Friday (beginning September 15), there will be a Quizzam/Quest during our regular class time. In other words, **Quizzam/Quest dates are Sept 15, Sept 29, Oct 13, Oct 27, Nov 13 (Monday!), and Dec 1.**

There will be no midterms in this class; you should think of the Quizzams as assessments which occur with the frequency of quizzes but the seriousness of tests/exams.

Most of the problems on the Quizzam will come from the homework assignment you received two weeks earlier. Some of the problems on the Quizzam will be taken verbatim from the

homework assignment, and others will be minor variations. One problem on each Quizzam might come, instead, from the WeBWorK or from the Tuesday worksheets.

If you fully understand how to solve the homework problems, you should have no difficulties with the Quizzam. Conversely, if you have not done the homework assignment, you will not have enough time to complete the Quizzam.

Just as on the homework, I expect you to **write legibly, logically, and in complete sentences** on the Quizzams. On the first Quizzam, as on the first homework, I will mark places where your writing does not meet these standards, but I will not take points off until the second Quizzam.

The Honor Code requires that on the Quizzams, as on the final exam, you will use no resources other than your brain, a pencil or pen, and a blank sheet of paper. Using (or providing another student with) other resources on the Quizzams or the final will result in a grade of zero for that assessment, and possibly additional sanctions.

Each of the six Quizzams will be worth 20 points; however, the “Quizzams” portion of your grade will be determined out of 100 points rather than 120. (You can think of this as dropping your lowest quiz score.) Thus, **each Quizzam will be worth 5% of your final grade**; a poor quiz score will drop a B+ grade to a B or B-. Please treat them accordingly.

Neither Quizzams nor the final exam may be taken late; if you have an unavoidable, University-approved conflict with a scheduled Quizzam or with the final, please let me know as soon as possible so we can schedule a time for you to take it early.

2.5.2 Homework presentations (12%)

On each written homework assignment, each group will be given one or two homework problems which they will be responsible for presenting to the rest of the class. The Wednesday before the Quizzam will be devoted to presentations of the homework problems.

As you come in to class on Wednesday, each person can vote for which homework problems they would like to see presented. The most popular ones, and the ones which I think are particularly important, will be presented at the board by a member of the group assigned to that problem. **This group member will be chosen at random.** After the presentation, the other group members will be responsible for answering questions about this problem from their classmates.

My goal with this presentation format is to ensure that everyone from the group has learned the material and gets a chance to practice their presentation skills. If you know the material well enough to teach it (to your groupmates or your classmates) you certainly know it well enough to do well on the Quizzams and the final exam!

The presentations will be graded on clarity and on their mathematical correctness, according to the rubric posted on Moodle. Each group should work together on solving your assigned homework problem and preparing the presentation: **the presentation grade will be the same for all members of the group.** It is your responsibility to make sure that everyone in your group understands the solution to your assigned problem(s), and can present the solution(s) clearly at the board. I recommend meeting with your group to practice your presentations thoroughly! We will also spend some time in class practicing presentation techniques, and discussing what makes a good presentation, in class on Wednesday September 6.

Each group will present on at least two of the three presentation Wednesdays. To enable this, **each presentation should take at most 5-7 minutes.**

Your total presentation grade for the first half of the course will be determined by the average of your group's presentation scores; your total presentation grade for the class will be the sum of your first group's average and your second group's average.

3 Course Policies

3.1 Asking and Answering Questions

It's important to me that our class is paced for everyone, not necessarily the pace of the most vocal students. I want to make sure that everyone's questions are heard. So, if you have a question, feel free to ask it at any time; if I have a question for the class, I will roll dice to determine which student I will ask. Please feel free to say "I don't know" – that tells me that I need to try a different explanation. Almost always, if you're confused, there are at least 3 other people in the same class with the same confusion!

3.2 Attendance

Since we will do so much group work in M 273, **your regular attendance – physical and mental** – is crucial not only to your own success in this course, but also to your classmates. Failure to attend class regularly will have a negative effect on your grade.

3.3 Student Classroom and Course-Related Behavior

University policy requires that all of us in the classroom treat each other with respect, and refrain from behavior that will disrupt the educational process. In particular, I expect that you will **focus your attention on Math 273 during class time!** For example, please refrain from cell phone use (including texting), computer use (unless you take notes on your computer), working on homework for another class, or talking about Friday night's awesome party. Our classroom is not the time or place for these activities, fun as they are. I know from experience how easy it is to think "I know this material, it's a good time to take 10 seconds and check my email" – only to surface a full minute later, having missed something important.

Please learn from my experience and don't let yourself get distracted during class. If it's really a 10-second email-check, you could do that just as easily after class. If you find that you're spending minutes at a time completely bored and disconnected from what we're doing in class, so that checking your email really would be a better use of those minutes, please come talk to me. That means I need to adjust how I'm structuring class time.

In my turn, I promise to treat all of you with respect. For example, if you would prefer me to call you by a different name, or gender pronoun, than is listed on the course roster, please let me know and I'll be happy to oblige.

3.4 Due dates and late work

Unless stated otherwise, all assignments are due at the beginning of class on the day on which they are due. Late work will receive no credit. Turning in an assignment 5 minutes after the start of class is acceptable (unless it becomes a habit); handing in an assignment 20 minutes late is **not**.

3.5 Disability Modifications

The University of Montana assures equal access to instruction through collaboration between students with disabilities, instructors, and Disability Services for Students. If you have a disability that adversely affects your academic performance, and you have not already registered with Disability Services, please contact Disability Services in Lommasson Center 154 or 406.243.2243. I will work with you and Disability Services to provide an appropriate modification.

3.6 Academic Honesty

All students must practice academic honesty. Academic misconduct is subject to an academic penalty by the course instructor and/or a disciplinary sanction by the University. Specific examples of how this applies to the various assessments in this course can be found in the section describing each assessment above.

3.7 Religious Holidays and Absences from Classes and/or Exams

If a religious observance conflicts with a scheduled exam or other course activity, please let me know **during the first two weeks of the semester** so that we can make alternate arrangements.

3.8 Student Conduct Code

All students need to be familiar with the Student Conduct Code. You can find it in the “A to Z” index on the UM home page.

In particular, discrimination and harassment are not tolerated at the University of Montana. If you feel that you have been subjected to discriminatory or harassing behavior, I’m very sorry to hear it; please contact the Office of Equal Opportunity and Affirmative Action at 243-5710 or <http://www.umt.edu/eo/equalop/harassment.php> for help in addressing the situation. You can also report the discrimination or harassment to me or to another faculty member you trust.