Math 555 - Functional Analysis

Fall 2017

Course Instructor: Dr. Elizabeth Gillaspy

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Lecture hours and Venue: MWF 11:00 a.m. - 11:50 a.m., MATH 206

Office Hours: Monday 1:30 - 2:30 p.m, and by appointment.

Course Prerequisites: Math 473 (Real Analysis) or Math 472 (Complex Analysis)

Textbook: John Conway, *A course in functional analysis.*

There is a copy of the textbook at the library, which I can place on reserve for the course if you would like. I will also be loosely following Stefaan Vaes' lecture notes on functional analysis, which I have posted on our course Moodle page.

1 Course Contents

Welcome to Math 555, Functional Analysis! My main goal with this class is to prepare you for the topics course on "Graph C^*-algebras" I will teach next semester. However, functional analysis is much broader than my research area of C^*-algebras. In addition to the various types of analysis that come under the umbrella of functional analysis, there are links and applications to algebra, logic, geometry, and topology as well. Please keep me informed of how your other mathematical interests relate to this course, and I'll do what I can to tailor the course to make it interesting and relevant to everyone.

To get there, we will start with the basics of Hilbert spaces and Banach spaces, their dual spaces, and some of the natural topologies on these spaces. We will prove the three main pillars of functional analysis: the Hahn-Banach Theorem, the Uniform Boundedness Principle, and the Open Mapping Theorem. Then we'll move on to questions about linear and continuous operators on Hilbert spaces. I hope to prove the Spectral Theorem for compact operators, and introduce C^*-algebras, before the end of the semester.

To do this, we will study Chapters I-III of Conway's text *A course in functional analysis* in detail, and then jump around in the text according to time constraints and the interests of the class. As mentioned, I would like to spend some time in Chapter VIII, but Chapter V also has a wealth of interesting material.

1.1 Learning outcomes

Upon completion of this course, a student will be able to:

1. Define central objects of study in functional analysis, such as Hilbert spaces, Banach spaces, Banach algebras, bounded linear operators, the weak-* topology, and the norm topology;

2. Distinguish between these objects, and give examples and non-examples of them;

3. Identify important properties of these objects, such as the Cauchy-Bunyakowski-Schwarz inequality, the parallelogram law, and Parseval's identity for Hilbert spaces;

4. Recognize when to apply fundamental results such as the Hahn-Banach Theorem, Uniform Boundedness Principle, Open Mapping Theorem, and Spectral Theorem for Compact Operators, in proving theorems or exercises.

Through this course, students will also improve their ability to read and communicate mathematics.

2 Course Structure

Your grade in this course will be based on:

- Reading assignments/Discussion posts (5%)
- Presentations (15%)
- Homework (80%)

2.1 Reading assignments/Discussion posts

In every class I teach, I want students to improve their ability to communicate math, not just to enhance their mathematical knowledge. One aspect of this is improving your ability to read math. To that end, I will usually ask you to read a section of the textbook before class, and post a comment in the relevant Discussion forum in Moodle by **10 AM** the morning before class. I don't expect you to be able to understand everything in the textbook at the beginning of the semester – this is the role of the Discussion forums, for you to tell me what you didn't understand from the textbook and what you want to focus on in class that day. However, I do expect you to put serious effort into the reading assignments, so that your ability to learn math from the textbook will improve over the course of the semester. We will spend time in class discussing strategies for reading a math textbook; one aspect of this is to **budget 10 minutes per page** for the reading assignments.

The discussion posts are a means for you to synthesize what you've learned by reading the textbook, connect the material to previous sections, and let me know what you still need some help with. Your comments in the Discussion forums will be graded on a scale of 0 to 2, based on how well you convince me that you have read the section and thought about it, and I will drop your lowest 5 scores.

For example:

- You will receive a 0 if you don't post any comment.
- A comment like "Everything makes sense, I have no questions" will earn a score of 0.

  (Even if everything makes sense, I'm sure you can find something more specific to say! How does this section connect with other sections? What did you think was interesting? Was there any part of the section that didn't make sense at first? What was it that eventually made that part make sense?)

- If your comment asks a question that indicates to me that you only skimmed the textbook, you will earn a score of 1.

  For example: "What does the author mean by $\{a_n\}$?"

  (This is explained in words on page 2 of the textbook.)

- A comment such as "I didn't remember the definition of a field, so I looked it up. Why do we need to work with fields instead of rings when defining an inner product?" will earn a score of 2.

  This question demonstrates that the author read the section and thought about what s/he read.

You are welcome to consult other textbooks for the reading assignments, if you prefer. To facilitate this, in each Moodle discussion forum, I will describe the topic of the reading so that you can find the appropriate material in another textbook.

If you do consult other textbooks, please be careful to use the notation from Conway's book on homework assignments and when posting in the Discussion forums, or to define the notation you use.

2.2 Presentations

To enhance the relevance of M 555 to your interests, I will ask each of you to give a short presentation (20-25 minutes) about a topic of interest to you that fits under the umbrella of Functional Analysis. You can base this presentation on a section of the textbook that we haven't covered in class, or on material from another textbook or a paper.

Please let me know as soon as you have an idea for a presentation topic. Then we can fine-tune the choice of topic and source materials together, and schedule your presentation so that it fits well with the arc of the course and with your own schedule. At the very latest, you should **meet with me before Thanksgiving** to confirm your presentation topic and schedule your presentation.

2.3 Homework

The majority of your grade will be based on the weekly written homework assignments, which will be due on Wednesdays at the beginning of class. Each of the 13 homework assignments will be graded out of 20 points, but the total number of points in the "Homework" category of your grade will be 200, to give you a buffer for busy weeks.

I encourage you to discuss the homework problems together, but **you must write up your solutions on your own**, and acknowledge, in your write-ups, any sources (human or otherwise) that you consulted, other than me or the course textbook. During discussions with classmates (or with me) about the homework problems, feel free to take a few notes, but you should not write up the solutions during this discussion period. Waiting to write up the solutions until you're in the privacy of your home or office confirms that the solutions you write up reflect your own understanding of the problem.

Indeed, when writing up your homework, please **consult your notes, and any other resources, as minimally as possible.** Before writing the proof, think through the argument. How do all the pieces fit together? What's the best order to explain things? Then, once you

start writing, do your best to write the solution without reference to your notes or the textbook. If you have to look up a reference more than 2 or 3 times during the writing, you don't fully understand the solution yet. If this happens, I suggest you stop working on the final draft; work through again (using notes) how the argument should go; and take a break, perhaps by moving on to another problem. When you come back to the tricky problem later, you should have a better, deeper understanding of how the proof works.

Failure to follow these guidelines may result in homework solutions that are uncomfortably close to plagiarism. Plagiarism (copying solutions from a source other than your own brain) constitutes academic misconduct, and will result in a loss of credit for the assignment and/or disciplinary sanctions.

3 Course Policies

3.1 Asking and Answering Questions

It's important to me that our class is paced for everyone, not necessarily the pace of the most vocal students. I want to make sure that everyone's questions are heard. So, if you have a question, feel free to ask it at any time; if I have a question for the class, I will roll dice to determine which student I will ask. Please feel free to say "I don't know;" that tells me that I need to try a different explanation. Almost always, if you're confused, there are at least one or two other people in the class with the same confusion!

3.2 Student Classroom and Course-Related Behavior

University policy requires that all of us in the classroom treat each other with respect, and refrain from behavior that will disrupt the educational process. In particular, I expect that you will **focus your attention on M 555 during class time!** For example, please refrain from cell phone use (including texting), computer use (unless you take notes on your computer), or working on homework for another class. I know from experience how easy it is to think "I know this material, it's a good time to take 10 seconds and check my email" – only to surface a full minute later, having missed something important.

In my turn, I promise to treat all of you with respect. For example, if you would prefer me to call you by a different name, or gender pronoun, than is listed on the course roster, please let me know and I'll be happy to oblige.

3.3 Due dates and late work

Unless stated otherwise, all assignments are due at the beginning of class on the day on which they are due. Late work will receive no credit. Turning in an assignment 5 minutes after the start of class is acceptable (unless it becomes a habit); handing in an assignment 20 minutes late is **not**.

3.4 Disability Modifications

The University of Montana assures equal access to instruction through collaboration between students with disabilities, instructors, and Disability Services for Students. If you have a disabil-

ity that adversely affects your academic performance, and you have not already registered with Disability Services, please contact Disability Services in Lommasson Center 154 or 406.243.2243. I will work with you and Disability Services to provide an appropriate modification.

3.5 Academic Honesty

All students must practice academic honesty. Academic misconduct is subject to an academic penalty by the course instructor and/or a disciplinary sanction by the University.

3.6 Religious Holidays and Absences from Classes and/or Exams

If a religious observance conflicts with a scheduled exam or other course activity, please let me know **during the first two weeks of the semester** so that we can make alternate arrangements.

3.7 Student Conduct Code

All students need to be familiar with the Student Conduct Code. You can find it in the "A to Z" index on the UM home page.

In particular, discrimination and harassment are not tolerated at the University of Montana. If you feel that you have been subjected to discriminatory or harassing behavior, I'm very sorry to hear it; please contact the Office of Equal Opportunity and Affirmative Action at 243-5710 or `http://www.umt.edu/eo/equalop/harassment.php` for help in addressing the situation. You can also report the discrimination or harassment to me or to another faculty member you trust.