UNIVERSITY OF MONTANA

Department of Mathematical Sciences

M 584
Extra-Combinatorial Methods

Course: M 584 Sec. 01 (CRN 33876) 3 cr., Spring 2017
Topics in C & O: Extra-Combinatorial Methods
TΘ 9:30–10:50am in MATH 211

Instructor: Mark Kayll **Econtact:** mark.kayll@umontana.edu
hs.umt.edu/math/people/default.php?s=Kayll
Office: MATH 209 **Hours:** M 2:00–2:50pm, Θ 10:50–11:40am & by appointment
406.243.2403 (*tentative*) (open for all course matters, including DSS accomm.)

Prerequisites: interest in the description below and mathematical maturity (at the level of a senior undergraduate or graduate student). Knowledge of basic linear and abstract algebra is desirable, as is some background in graph theory and/or combinatorics.

Text: László Babai and Péter Frankl, *Linear Algebra Methods in Combinatorics*, Preliminary Version 2 (September 1992), Dept. of Computer Science, University of Chicago (paperback, $30 plus shipping)

Important Dates:

last day to add by Cyberbear	Tuesday, 31 January (5pm);
last day to drop by Cyberbear, or select Audit grade option	Friday, 10 February (5pm);
Presidents' Day holiday	Monday, 20 February;
spring break	20–24 March;
Math Awareness Month	April (watch for events);
last day to add/drop by paper form	Monday, 3 April (5pm);
last day to add/drop by petition	Friday, 5 May (5pm);
last class meeting (during finals)	Monday, 8 May 8:00–10:00am.

Description: Most of mathematics is built from set theory. Since the world is finite[1], finite sets play a prominent role in our subject. Thus, combinatorics—the mathematics of finite sets—enjoys many interesting applications and connections to other mathematical branches. This course will explore some of these connections: to linear algebra, abstract algebra, probability, number theory, geometry, and analysis.

As examples, here are some results to consider. The Matrix Tree Theorem expresses the number of spanning trees of a graph G as the determinant of a matrix associated with G. On a more analytic bent, Godsil showed that the number of perfect matchings of G is

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} P(x) e^{-x^2/2}\, dx,$$

where $P(x)$ is a polynomial determined by G. Prefer abstract algebra?... Frucht's Theorem says that every finite group is the automorphism group of some graph. For a final example, we turn to extremal set theory: *How many subsets of an n-set can have pairwise the same intersection size* (a question related to statistical design theory)? Surprisingly, the answer is delivered by the basic fact that an N-dimensional vector space never contains more than N linearly independent vectors.

This course will explore these sorts of connections between discrete math and its 'outside world'. By considering a wide range of methods, we hope to instill a sense of connectedness of the mathematical enterprise.

(over)

Assessment: Grades are based on performance on items to be discussed in class, e.g., attendance, homework, and a presentation (not exams). Traditional letter grades will be assigned using the +/− system (see *UM catalog* at `www.umt.edu/catalog/academics/academic-policy-procedure2.php`). UM's policy on Incomplete grades will be followed (see *UM catalog*).

Homework: Details will be discussed in class.

Presentations: Each presentation consists of a 50-minute lecture scheduled during a regular, or final, class meeting. The content of the presentations should be related to the course content and may be inspired by one or more sections of the text, by related paper(s), or by other related material. Students should schedule their lecture date privately with the instructor early in the semester; time slots are assigned on a first-come, first-served basis. Lecture topics must be approved by the instructor, and students should take the following preparatory steps.

Step	Timing	Action
0	early in semester	Schedule lecture date with instructor.
1	3 weeks prior to lecture	Submit a $\leq$ 1-page typed summary proposal of lecture topic, with references.
2	2 weeks prior to lecture	Receive proposal approval or suggested modifications from instructor.
3	1 week prior to lecture	Meet with instructor privately for final informal discussion of lecture topic; be prepared to field questions.

Accommodation: The University of Montana assures equal access to instruction through collaboration between students with disabilities, instructors, and Disability Services for Students. If you have a disability that adversely affects your academic performance, and you have not already registered with Disability Services, please contact Disability Services in Lommasson Center 154 or 406.243.2243. The instructor will work with you and Disability Services to provide an appropriate modification.

General Remarks

On homework: Please use complete sentences, proofread, and polish your work prior to submission. You're encouraged to type homework solutions unless your handwriting is clear. You may work with others on homework problems, and you're encouraged to do so; however,

Solutions should be written down privately in your own words.

If you use an important idea of someone else, then please acknowledge that person by giving an appropriate citation in your write-up. This professional courtesy will not affect your grade.

On make-ups: There are no homework make-ups; this policy will be elaborated in class.

On deadlines: Any stated deadlines are firm; please do not ask for extensions. (Violating this request is considered grounds for a penalty on the corresponding assignment.)

On electronic devices: Cell phones must be silenced during class meetings and office hour visits.

On conduct: All students need to be familiar with the Student Conduct Code; it can be found in the 'A to Z Index' on the UM home page. All students must practice academic honesty. Academic misconduct is subject to an academic penalty by the course instructor and/or a disciplinary sanction by the university.

Additional References

1. N. Biggs, *Algebraic Graph Theory, 2nd edition*, Cambridge University Press, New York, 1993
2. G. Chartrand, L. Lesniak, and P. Zhang, *Graphs & Digraphs*, 6th edition, Chapman and Hall/CRC, Boca Raton, 2016
3. C.D. Godsil, *Algebraic Combinatorics*, Chapman & Hall, New York, 1993
4. C.D. Godsil and G. Royle, *Algebraic Graph Theory*, Springer, New York, 2001

[1]Combinatorics is the most fundamental, and hence the most important, branch of mathematics, since it deals with FINITE structures, and the world is finite.

Doron Zeilberger, *Board of Governors Professor of Mathematics*
Rutgers University